ORIGINAL ARTICLE

Numerical simulation of shock and detonation waves in bubbly liquids

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Abstract
A well-posed mathematical model of nonisothermal, two-phase, two-velocity flow of a bubbly medium with chemically inert liquid and chemically inert or active gas bubbles is proposed. The model is based on the two-phase Euler equations with the introduction of an additional pressure at the gas-bubble surface, which ensures the well-posedness of the Cauchy problem for a system of governing equations, and the Rayleigh–Plesset equation for radial pulsation of gas bubbles allowing solutions of the soliton type that are realized in experiments to be obtained. The model is validated by comparing calculations of detonation wave propagation with experimental data in water or an aqueous solution of glycerin with bubbles of acetylene–oxygen mixture or with bubbles of argon–hydrogen–oxygen mixture for a volume gas content of 1–6%. The model is shown to provide satisfactory results for the detonation propagation velocity and detonation pressure profiles.

1 Introduction

In [1–4], a propulsion device of a new type, pulsed detonation hydroramjets, for both water and underwater transportation was proposed. This propulsion device can operate in both pulsed and continuous detonation modes. In order to estimate the efficiency of the device and its design, it is necessary to understand the specific features of the two-phase liquid–gas flow with chemically inert and/or active gas bubbles.

Experimental studies of shock wave propagation in chemically inert bubbly flows were conducted in a number of works [5–16]. However, the case of the two-phase flow of liquid with chemically active gas bubbles is less studied [17–24].

In [17–24], it was shown that bubbly detonation, which is a self-sustaining detonation-like wave propagating in a quasistationary way at a supersonic speed, can occur during the penetration of a shock wave into a liquid containing chemically active equally spaced gaseous bubbles.

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Theoretical studies of bubbly detonation in the one-dimensional case are presented in [25–30]. In [31–33], bubbly detonations are studied in the two-dimensional approximation. In [25–28,31–33], a simplified model of two-phase flow has been considered. In these works, it is assumed that (i) the gas volume fraction is small, (ii) the gas obeys the ideal gas law, (iii) the liquid is incompressible (except [31–33]) and isothermal, (iv) the gas bubbles retain a spherical form, do not interact with each other, and do not disintegrate, (v) the radial oscillations of bubbles in the bubbly medium are described by the Rayleigh–Plesset equation, and (vi) the gas and liquid move with the same velocities.

The models [25–28,31–33] are attractive for their simplicity. However, the velocities of the gas and liquid differ significantly directly behind a shock wave, i.e., condition (vi) does not hold. In [29], a stationary two-phase two-velocity model of the bubbly flow was used. In our previous work [30], a two-phase two-velocity viscous model of the bubbly flow was used, but radial oscillations of the gas bubbles were not taken into account.

In the present work, we propose a new nonstationary two-velocity model of a two-phase flow to study the bubbly shock and detonation waves in a chemically inert liquid with chemically active or inert gas bubbles. This model is based on spatial averaging of the governing equations under conditions (i)–(v). Pressures in the gas and liquid phases are connected via the Rayleigh–Plesset equation. Also, a

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physically justified interfacial pressure is included in the governing equations to ensure well-posedness of the Cauchy problem [34,35].

2 Mathematical model

Let us consider a model for a nonisothermal, two-phase, two-velocity bubbly medium composed of chemically inert liquid and chemically inert and/or active gas bubbles:

$$
\begin{aligned}
&\frac{\partial \phi_f \rho_f}{\partial t} + \nabla_k(\phi_f \rho_f u_f^k) = 0 \\
&\frac{\partial \phi_f \rho_f u_f^j}{\partial t} + \nabla_k(\phi_f \rho_f u_f^k u_f^j + p_f \phi_f) - p_i \nabla_j \phi_f = F_f^j \\
&\frac{\partial \phi_2 \rho_2 Y_l}{\partial t} + \nabla_k(\phi_2 \rho_2 u_2^k Y_l) = \phi_2 \dot{\omega}_l \\
&\frac{\partial \phi_2 \rho_2 E_2}{\partial t} + \nabla_k(\phi_2 u_2^k(\rho_2 E_2 + p_2)) + p_i \frac{\partial \phi_2}{\partial t} \\
&\quad = \phi_2 \dot{\omega}_T + H_2
\end{aligned}
\tag{1}
$$

where index f denotes either the liquid phase (1) or the gas phase (2), ϕ_f, p_f, ρ_f, and E_f refer to the volume fraction, pressure, density, and total energy of phase f, respectively, $\mathbf{u}_f = u_f^j \mathbf{e}_j$ is the velocity of phase f, $\mathbf{e}_j$ is the covariant basis of the Euclidean space, and Y_l is the mass fraction of the component l in the gas phase.

It should be noted that in the system of (1) the law of energy conservation for the liquid is not written according to assumption (iii). The temperature change in the liquid due to chemical reactions in the gas bubbles can be estimated by the relation:

$$
\Delta T_1 = \frac{\rho_2^0 \phi_2^0 Q}{\rho_1^0 \phi_1^0 c_{p,1}}
\tag{2}
$$

where T_1 is the liquid temperature, index 0 denotes the initial state of the bubbly flow, Q is the heat effect of gas combustion, and $c_{p,1}$ is the specific heat of the liquid at constant pressure. For example, for water with bubbles of acetylene–oxygen mixture at $\phi_2^0 = 0.06$, $\Delta T_1 \approx 1$ K, i.e., water can be considered isothermal.

In the case of inert gas bubbles, the chemical reaction does not occur, i.e., the chemical reaction rate $\dot{\omega}_l = 0$ for each component l and the rate of energy release $\dot{\omega}_T = 0$. In the case of chemically active gas bubbles, the rate of energy release is defined as follows:

$$
\dot{\omega}_T = -\sum_{l=1}^{L} \Delta e_{2,l}^0 \dot{\omega}_l
$$

where L is the number of components in the gas phase, $\Delta e_{2,l}^0 = \Delta h_{2,l}^0 - \frac{RT_0}{W_l}$ is the formation standard energy of the component l, $\Delta h_{2,l}^0$ is the formation standard enthalpy of the component l, T_0 is the reference temperature, W_l is the molecular mass of the component l, and R is the universal gas constant. The rate of chemical reaction in the gas phase is defined as:

$$
\dot{\omega}_l = W_l \sum_{k=1}^{M} \left(v_{l,k}'' - v_{l,k}' \right) f_k.
\tag{3}
$$

Here M is the total number of chemical reactions, $v_{l,k}'' \left(v_{l,k}' \right)$ is the stoichiometric coefficient of the component l that is a product (reactant) in the chemical reaction k, and

$$
f_k = A_k T^{\alpha_k} e^{-\frac{E_k}{RT}} \prod_{j=1}^{L} \left(\frac{Y_j \rho_2}{W_j} \right)^{m_{j,k}}
\tag{4}
$$

where A_k is the preexponential factor, α_k is the temperature exponent, E_k is the activation energy of the chemical reaction k, and $m_{j,k}$ is the order of the chemical reaction k with respect to the component j (if not marked separately $m_{j,k} = v_{j,k}'$).

The interfacial pressure p_i is defined as suggested in [34,36]:

$$
p_i = p_1 + \frac{1}{2} C_s(\phi_2, \mathrm{Re}_{12}) \rho_1 \mathbf{u}_{12}^2
\tag{5}
$$

where $\mathbf{u}_{12}$ is the phase slip velocity and $C_s(\phi_2, \mathrm{Re}_{12})$ is the empirical function depending on the volume fraction and the Reynolds number of the relative phase motion $\mathrm{Re}_{12} = \frac{d\rho_1 |\mathbf{u}_{12}|}{\mu_1}$ (d is the mean diameter of bubbles in an elementary volume and μ_1 is the liquid viscosity). The function $C_s(\phi_2, \mathrm{Re}_{12})$ is defined as:

$$
C_s(\phi_2, \mathrm{Re}_{12}) = \alpha(\mathrm{Re}_{12}) C_{dp}^{coll}(\phi_2, \mathrm{Re}_{12})
\tag{6}
$$

where C_{dp}^{coll} is the shape drag coefficient taking into account collective effects:

$$
C_{dp}^{coll}(\phi_2, \mathrm{Re}_{12}) = C_{dp}(\mathrm{Re}_{12}) (1 - \phi_2)^{-2.7}.
\tag{7}
$$

The coefficient $\alpha(\mathrm{Re}_{12})$ in (6) and the function $C_{dp}(\mathrm{Re}_{12})$ in (7) are linearly interpolated according to the values presented in Table 1.

The sum of volume fractions is equal to unity:

$$
\phi_1 + \phi_2 = 1.
\tag{8}
$$

Table 1 Coefficients C_{dp} and α depending on the Reynolds number Re_{12}

Re_{12}	α	C_{dp}
10	-0.13	1.5
100	-0.52	0.48
400	-0.96	0.32
1000	-0.74	0.43
163000	-0.74	0.43

The phase pressures are connected via the Rayleigh–Plesset equation describing pulsations of a gas bubble:

$$
\frac{d_2 R}{dt} = \omega_b
$$
$$
\rho_1 R \frac{d_2 \omega_b}{dt} + \rho_1 \frac{3}{2} \omega_b^2 = p_2 - \frac{2\sigma}{R} - p_1 - \frac{4\mu_1 \omega_b}{R} \tag{9}
$$

i.e., the pulsations of the gas bubble are determined by the inertia of the added mass, the difference in phase pressures, and viscous dissipation. In (9), $R = \dfrac{d}{2}$ is the mean radius of gas bubbles in an elementary volume, ω_b is the mean velocity of gas-bubble pulsation, and $\dfrac{d_2}{dt} = \dfrac{\partial}{\partial t} + u_2^k \nabla_k$ is the substantial derivative. The mean radius of gas bubbles and the volume fraction are related by the relationship:

$$
\phi_2 = \frac{4}{3} \pi R^3 N \tag{10}
$$

where $N = $ const is the number of gas bubbles in an elementary volume.

According to assumptions (ii) and (iii) (see Sect. 1), the phase equations of state have the following form:

$$
\rho_1 = \rho_1^0 = \text{const}, \quad \rho_2 = \frac{p_2 W}{R T_2} \tag{11}
$$

where W is the molecular mass of the mixture:

$$
\frac{1}{W} = \sum_{l=1}^{L} \frac{Y_l}{W_l}.
$$

According to assumption (iii), the temperature of the carrier liquid $T_1 = T_1^0 = $ const, and T_2 is defined by the caloric equation of state:

$$
e_2 = \int_{T_2^0}^{T_2} c_{v,2}(T_2) dT_2
$$

where $c_{v,2}(T_2)$ is the specific heat at constant volume [37].

The system of (1)–(11) is supplemented by the relations for the interfacial energy and momentum exchange:

$$
F_2^j = -F_1^j = C_d(\text{Re}_{12}) \frac{A \rho_1 (u_1^j - u_2^j)|u_1^j - u_2^j|}{8}
$$
$$
H_2 = \text{Nu}_2 \lambda_2 \frac{A(T_1 - T_2)}{d} \tag{12}
$$

where T_f is the temperature of phase f, $A = \dfrac{6\phi_2}{d}$ is the interfacial area of the bubbly liquid in a unit volume, λ_2 is the thermal conductivity coefficient of the gas, and C_d is the hydrodynamic drag coefficient of a single spherical bubble.

For relatively small Reynolds numbers ($\text{Re}_{12} < 100$), a gas bubble retains a spherical form and the drag coefficient is the same as for the rigid sphere. At moderate Reynolds numbers, a gas bubble takes the shape of an ellipsoid of revolution flattened in the direction of motion. The range of the Reynolds numbers in which gas bubbles retain the shape of an ellipsoid depends, in general, on the properties of the carrier liquid. In the present model, the coefficient C_d for such an ellipsoidal bubble was approximated by the values experimentally measured for air bubbles in water [38]. With further increase in the Reynolds number, the gas bubbles take the shape of a disk whose drag coefficient is independent of its size and is approximately equal to the constant value $C_d \approx 2.6$.

In this work, the deformation of gas bubbles is taken into account only in the value of coefficient C_d, and the effective interphase surface A in (12) is calculated, for simplicity, under the assumption that the gas bubble has a spherical shape.

The Nusselt number of the gas phase Nu_2 is defined as follows [39]:

$$
\text{Nu}_2 = \begin{cases} \sqrt{\text{Pe}_2}, & \text{Pe}_2 \geq 100 \\ 10, & \text{Pe}_2 < 100 \end{cases} \tag{13}
$$

where $\text{Pe}_2 = 12(\gamma_2 - 1) \dfrac{T_1 R |\omega_b|}{|T_2 - T_1| \varkappa_2}$, $\varkappa_2 = \dfrac{\lambda_2}{\rho_2 c_{p,2}}$ is the thermal diffusivity of the gas phase, γ_2 is the ratio of specific heats, $c_{p,2}$ is the specific heat at constant pressure, and λ_2 is the thermal conductivity. These coefficients depend on the temperature of the multicomponent gas [37].

It is worth noting that (13) is true, strictly speaking, only for weak shock waves (SW). Nevertheless we applied it as the first approximation to strong SWs and verified the validity of this approximation by changing the Nusselt number of (13) by a factor of 2 (see below).

The applicability of (1)–(13) to the modeling of SW propagation in a liquid with chemically inert or active gas bubbles is verified by comparison with experiments [13,19,22,23]. In these experiments, water or a solution of water and glycerin or silicone oil is used as a liquid phase and

Table 2 Parameters of the OKM for modeling self-ignition of the stoichiometric acetylene–oxygen mixture in the ranges of initial pressure 30 atm $< p_2^0 < 150$ atm and initial temperature 1500 K $< T_2^0 < 2500$ K

Reaction	A m, mol, s	α	E kJ/mol
$C_2H_2 + 1.5O_2 \longrightarrow 2CO + H_2O^1$	10^{11}	0	55.4
$CO + 0.5O_2 = CO_2$	2×10^9	0	12
$H_2O + M = H + OH + M$	1.6×10^{27}	-3.0	124

[1] $m_{C_2H_2} = 0.5$, $m_{O_2} = 0.5$
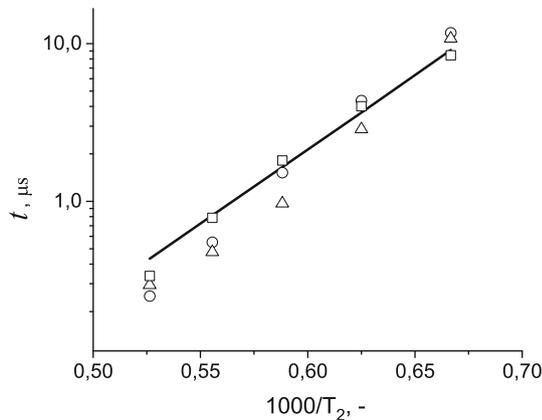
Fig. 1 Dependence of the self-ignition delay of the stoichiometric acetylene–oxygen mixture on the temperature: symbols—DKM [37], curve—OKM; triangles—$p_2^0 = 30$ atm, circles—$p_2^0 = 70$ atm, squares—$p_2^0 = 150$ atm

nitrogen or acetylene–oxygen or argon–hydrogen–oxygen mixture is used as a gas phase. An overall kinetic mechanism (OKM) is used for modeling chemical reactions in the acetylene–oxygen mixture (see Table 2). The kinetic parameters E_k, α_k, A_k, and $m_{j,k}$ in (4) were estimated by the self-ignition delay of the acetylene–oxygen mixture within the appropriate range of temperature and pressure (see Fig. 1). The self-ignition delay was calculated on the basis of a detailed kinetic mechanism (DKM) [37].

The isothermal sound velocity in bubbly liquid c_T can be estimated as follows [40]:

$$c_T^2 = \frac{p_1}{\rho_1 \phi_1 \phi_2}. \tag{14}$$

For the proposed model of (1)–(13), the sound velocity was not obtained analytically due to the difficulty in finding model eigenvalues. However, the sound velocity is in fact the propagation velocity of weak disturbances and a particular numerical experiment was performed to determine it. In the numerical experiment, we considered a mixture of silicone oil with nitrogen bubbles with the gas volume fraction ranging from 0.15 to 10%, and a weak disturbance was set by $\Delta p_1 = 1$ kPa. Figure 2 shows a good agreement of the sound
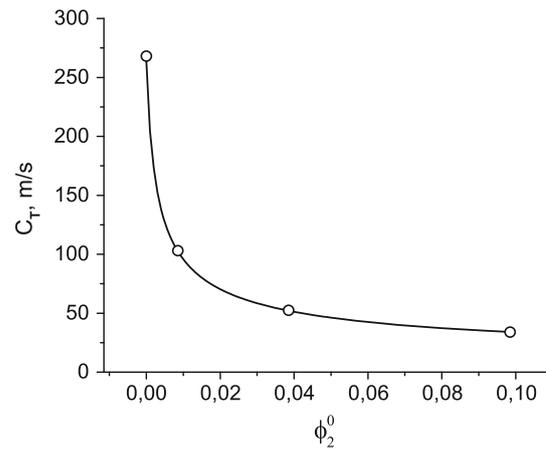
Fig. 2 Dependence of the isothermal sound velocity in silicone oil with nitrogen bubbles on the initial gas volume fraction: Circles correspond to the calculation based on (1)–(13) and solid line corresponds to (14)

velocity based on the proposed model and the sound velocity estimated by (14). Excellent agreement is worth mentioning.

3 Results and discussion

The numerical method for solving the governing equations (1)–(13) is based on the method of splitting by physical processes. First, the kinetic equations for the chemical transformations at constant volume are solved by Cantera, the open-source suite of tools for problems involving chemical kinetics, thermodynamics, and transport processes [41]. Thereafter, the system of (1)–(13) is solved without chemical sources by the method proposed in [13], which was slightly modified.

The kinetic equations for chemical transformations are described by the initial value problem of stiff ordinary differential equations which we write in the general form:

$$\dot{y} = f(y, t)$$
$$y(t_0) = y_0 \tag{15}$$

where $y \in \mathbb{R}^n$.

The system of (15) is solved by the variable order and variable time step BDF method (backward differentiation formula):

$$\sum_{i=0}^{q} a_{n,i} y^{n-i} + h_n \dot{y}_n = 0$$
$$\dot{y}_n = f(y_n, t_n). \tag{16}$$

Here y^n are the computed approximations of $y(t_n)$, and q is the approximation order varying from 1 to 5, $h_n = t_n - t_{n-1}$. Coefficients $a_{n,i}$ are uniquely determined by the
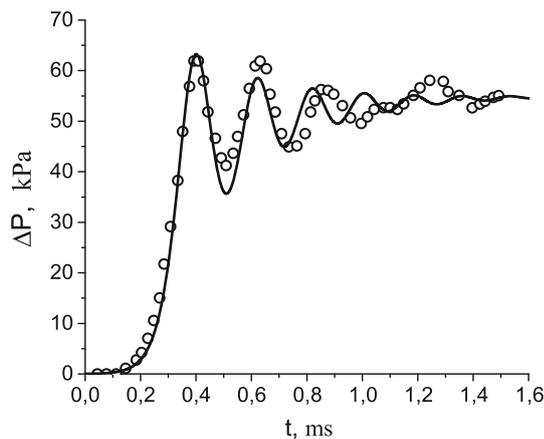
Fig. 3 Comparison of the experimentally measured (symbols [13]) and calculated (curves) pressure profiles in a bubbly SW; pressure sensor is installed at a depth of 314 mm below the contact surface between gas and bubbly medium

order, the recent history of the time steps, and the normalization $a_{n,0} = -1$. The selection procedure of the approximation order and time step is discussed in detail in [42].

For model validation, we first compared the results of the calculations with the experimental data [13] (chemically inert liquid with chemically inert gas bubbles) and then with the experimental data [19,22,23] (chemically inert liquid with chemically active gas bubbles).

The experiments [13] were carried out in a vertical shock tube with an inner diameter of 18 mm. The shock tube consisted of a high-pressure section (HPS—1.6 m), low-pressure section (LPS—1.8 m), and measuring section (MS—2 m) filled by silicone oil (KF96-50) with nitrogen bubbles at standard atmospheric conditions. We compared the results of numerical calculation with the experimental data [13] for the structure of a SW propagating in an inert bubbly liquid.

Figure 3 compares the experimental data [13] with the calculation results for a pressure profile in a SW propagating in the inert bubbly liquid at the conditions of the experiment: $\phi_2^0 = 0.0015$, $p_1^0 = 103.1$ kPa, $\Delta p_1 = 55.6$ kPa, and $R^0 = 0.606$ mm. It is seen that the calculation curve fits both qualitatively and quantitatively well with the measurement in terms of the frequency (the error is less than $\approx 13\%$) and amplitude (the error is less than $\approx 10\%$) of the pressure oscillations.

Figure 4 depicts the experimental pressure records for the detonation wave propagating in an inert liquid with chemically active gas bubbles [23]. The experiments [23] were carried out in a vertical shock tube with an inner diameter of 35 mm. The shock tube consisted of a HPS—1.15 m, LPS—0.29 m, and MS—4 m filled by an aqueous solution of glycerin (85%) with gas bubbles of 70%Ar+30%(2H$_2$+O$_2$). In the bubbly liquid, a specific perturbation is formed: A short-duration solitary wave (1) is separated from the SW (2), while SW (2) and SW (3), reflected from the end of the shock

Fig. 4 Pressure records demonstrating the formation and propagation of bubbly detonation [23]; sensor 1 is installed in the gas (**a**), sensor 2 is installed in the bubbly liquid at the depth of 285 mm (**b**), sensor 3—815 mm (**c**), sensor 4—1345 mm (**d**), where depth is counted from the interface between gas and bubbly medium; $\phi_2^0 = 0.0035$, $d^0 = 2.8$ mm, gas bubbles— 30%(2H$_2$ + O$_2$) + 70%Ar, carrier liquid–water
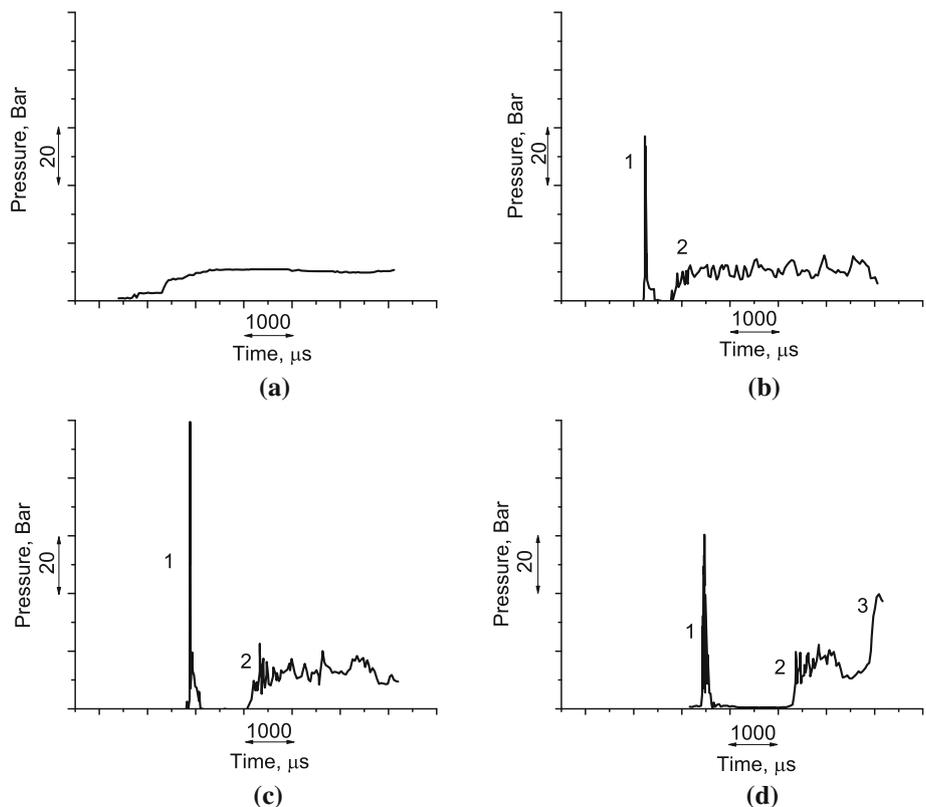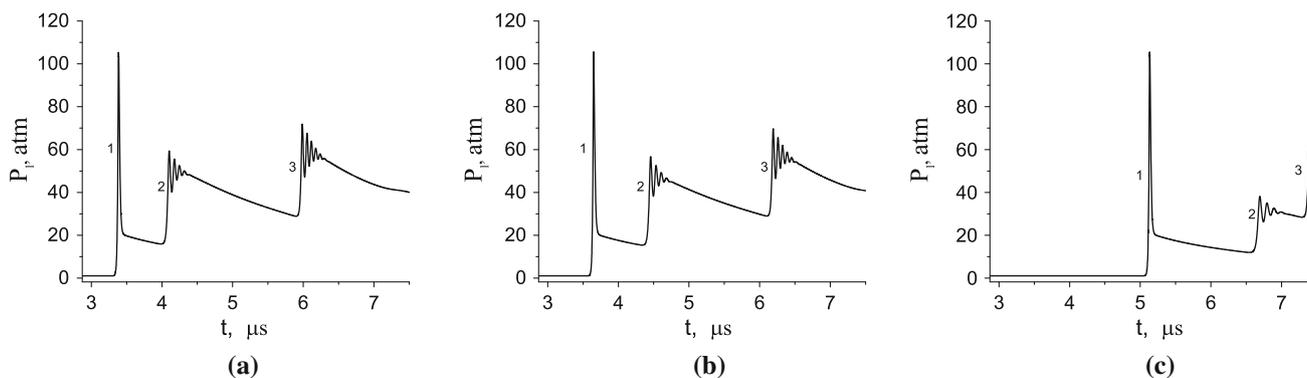
Fig. 5 Calculated pressure profiles in a SW propagating in water with bubbles of the stoichiometric acetylene–oxygen mixture. Sensor 1 is installed at a depth of 2340 mm, 2—2540 mm, 3—3730 mm, $\phi_2^0 = 0.01$, $d^0 = 3.5$ mm, system I; 1—soliton, 2—SW, 3—reflected SW

tube, lag behind the solitary wave (see records of pressure sensors 2, 3, and 4 in Fig. 4). According to the records of photosensors, ignition of gas bubbles occurs only in the solitary wave (1), while there is no ignition in the SW (2) and SW (3). An analysis of the solitary wave (1) motion shows that with time, this wave attains the quasistationary regime of propagation, which depends on the conditions of experiment, and propagates with a velocity, which significantly exceeds the equilibrium sound velocity in the bubbly liquid [43]. The velocity of the SW (2) following the solitary wave (1) decays due to energy dissipation. Such a stationary supersonic self-sustaining reaction front with rapid and complete fuel burnout in the solitary wave is referred to as *bubbly detonation*.

According to [20], the mechanism of bubbly detonation formation can be the following. It is known that in bubbly media with inert gas bubbles, a SW with an oscillatory structure can exist. This oscillatory SW can decay into a sequence of solitary waves—the solitons. The velocities of the solitons are proportional to their amplitudes. The presence of chemical reactions leads to an increase in the pressure in the leading front of the wave and hence to an increase in its propagation velocity.

In order to validate the model, we compared the results of the numerical calculation with the experimental data [19,22,23] for the structure of the bubbly detonation wave and its propagation velocity in a bubbly medium. As a bubbly medium, three systems were considered: water with bubbles of the acetylene–oxygen mixture (system I), aqueous solution of glycerin with bubbles of the acetylene–oxygen mixture (system II), and aqueous solution of glycerin with bubbles of the argon–hydrogen–oxygen mixture (system III).

Experiments [19,22] were carried out in a vertical shock tube with an inner diameter of 35 mm. The initial pressure and temperature (62.94 atm and 4278 K) in the HPS were created by burning the stoichiometric acetylene–oxygen mixture. The LPS was filled with air at standard conditions.
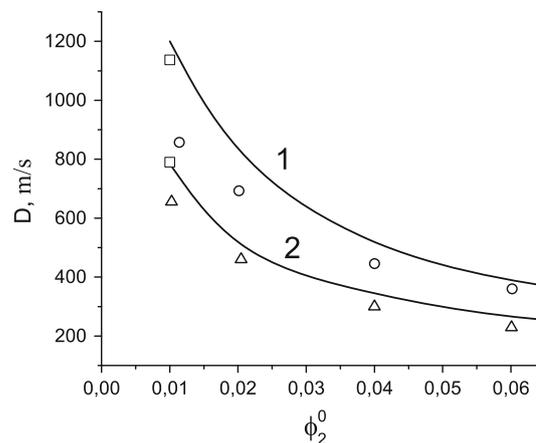

Fig. 6 Comparison of experimentally measured (circles and triangles [19]) and calculated velocities of solitons (1) and SW (2) in water with bubbles of the gaseous stoichiometric acetylene–oxygen mixture as a function of the initial gas volume fraction; curves are plotted based on the calculations with OKM, and squares correspond to the calculation with DKM

The measuring section was filled with the bubbly medium (system I or II) with the initial gas volume fraction ϕ_2^0 ranging from 0.01 to 0.1 and the mean gas-bubble diameter of $d^0 \approx 3.5–4.0$ mm (mass fractions of C_2H_2 and O_2 were equal to 0.245 and 0.755, respectively) at the standard conditions.

Figure 5 shows the calculated pressure wave profiles at the conditions of the experiment [19]. As in the experiment, in the calculation the formation of a soliton (1) and SW (2) is observed. In Fig. 6, the measured mean velocities of the soliton (1) and SW (2) are presented by circles and triangles as a function of the initial gas volume fraction in system I. In [19], the mean velocities were obtained using the records of the sensors installed at depths of 3530 and 3570 mm.

Note that in [19], bubbly detonation with the observed glow of the combustion products of acetylene–oxygen mixture occurred only within the range of ϕ_2^0 from 0.02 to 0.06,
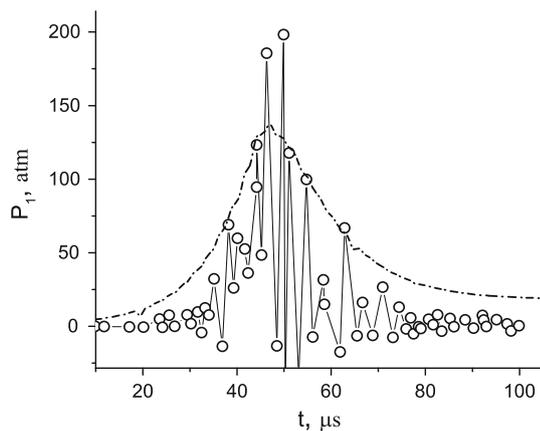
Fig. 7 Comparison of experimentally measured (circles [22]) and calculated (curve) pressure profiles in the soliton (system II, $\phi_2^0 = 0.01$, $D \approx 1130$ m/s, $p_1^0 = 1$ atm)

Fig. 8 Comparison of experimentally measured (circles [23]) and calculated with DKM (curve) pressure profiles in the soliton (system III, $\phi_2^0 = 0.004$, $D \approx 780$ m/s, $p_1^0 = 1$ atm)

while at $\phi_2^0 > 0.06$ the glow of the combustion products of acetylene–oxygen mixture was not observed, i.e., there was no bubbly detonation. Figure 6 shows that the results of calculations agree satisfactorily with the experimental data.

As mentioned above, we conducted two numerical experiments to check the sensitivity of the detonation velocity to the interface heat exchange. In the first experiment, the interphase heat exchange was calculated using (12), (13) and the detonation velocity at $\phi_2^0 = 0.01$ was equal to 1200 m/s (see Fig. 6). In the second experiment, the Nusselt number was increased by a factor of two and the detonation velocity was changed to 1130 m/s, which is 6% lower than in the first experiment. This means that the sensitivity of the detonation velocity to the value of Nusselt number is low.

The measured and calculated velocities of the solitons significantly exceed the equilibrium sound velocity in water with gas bubbles at all initial gas volume fractions. The frozen sound velocity c_f can be estimated as follows [40]:

$$c_f^2 = \frac{\gamma p_1^0}{\rho_1 \phi_1^0 \phi_2^0} \tag{17}$$

where $\gamma \approx 1.33$ for the $C_2H_2 + 2.5O_2$ mixture. The sound velocity was approximately 82 m/s at $\phi_2^0 = 0.02$ and 48 m/s at $\phi_2^0 = 0.06$ under the conditions of the experiment [19], i.e., at these conditions the soliton propagated with the Mach number equal to 9.4 and 8, respectively.

Figure 7 compares the measured [22] and calculated pressure profiles in the soliton which propagates in a quasistationary manner with the velocity $D \approx 1130$ m/s in system II at $\phi_2^0 = 0.01$ and $p_1^0 = 1$ atm. It is seen that both in the experiment and in the calculation, the half-width of the pressure wave is virtually the same and is about 30 μs.

The experiment in [23] was carried out in a vertical shock tube with geometric characteristics the same as in experi-
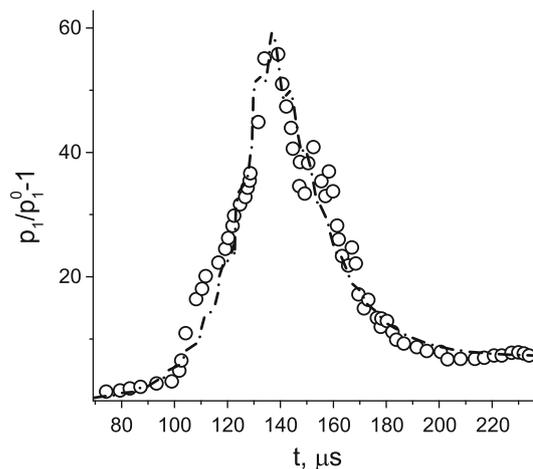
ments [19,22]. The measuring section was filled with the bubbly medium (system III) with the initial gas volume fraction ϕ_2^0 of 0.004 and the mean gas-bubble diameter d^0 of 1.4 mm (mass fractions of Ar, H_2, and O_2 were equal to 0.89, 0.01, and 0.1, respectively) at the standard conditions. Comparison with the experimental profile (see Fig. 8) shows good agreement with respect to amplitude and half-width.

4 Conclusions

In this work, a new mathematical model of a nonisothermal, two-phase, two-velocity flow of a bubbly medium with chemically inert liquid and chemically active gas bubbles is proposed. This model can be used to calculate the propagation of shock and detonation waves in bubbly media.

It is shown that in a bubbly medium, which consists of an inert liquid and chemically active gas bubbles, a quasistationary supersonic self-sustaining reaction front can propagate with a rapid and complete fuel burnout in the leading wave. Such a reaction front can be treated as detonation-like or as a bubbly detonation.

The model can be used to calculate two-phase reactive flows in a new type of propulsion device—pulsed detonation hydroramjets.

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