NON-REFLECTING BOUNDARY CONDITIONS AT OPEN BOUNDARIES FOR MODELING MULTI-DIMENSIONAL REACTIVE FLOWS

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INTRODUCTION

When simulating the multi-dimensional flow in a combustor, non-physical reflections are to be avoided at open boundaries, and transparency to acoustical waves must be ensured. We are addressing a basic practical issue in uniquely determining a numerical solution of unsteady subsonic flows. It is related to the solution of flow equations and involves choosing appropriate non-local boundary conditions at the inlet/outlet boundaries which will adequately bound a computational domain. A number of examples of successful implementation of the new boundary conditions will be presented.

APPROACH

It is assumed that to the left of the outlet boundary, $z=0$, there exist a stationary solution $p=p_0$, $\rho=\rho_0$, $u=u_0$, where p_0, ρ_0, and u_0 are the constant pressure, density and velocity. For the sake of simplicity, flow velocity has a single non-zero component, u_0 along the z axis. The flow is assumed subsonic, i.e. $M = u_0/c_0 < 1$, where c_0 is the sound speed. Let us linearize the problem in the vicinity of the stationary solution by representing the solution in the form

$$p = p_0 + p',\quad \rho = \rho_0 + \rho',\quad u = u_0 + u',$$

where p', ρ', u' are the variations of the corresponding parameters in the sound wave ($p' \le p_0, \rho' \le \rho_0, u' \le u_0$). Standard transformation of Euler equations results in the equation for p' («prime» is removed):

$$p_{tt} + 2u_0 p_{tz} + u_0^2 p_{zz} = C_0^2 \Delta p .$$

Assume that the geometry of the channel allows for the separation of variables, i.e.

$$p = Y(z,t) f(r) g(\varphi) .$$

Applying the Riemann method to the equation for $Y(z,t)$, followed by the Laplace transformation and standard techniques for taking contour integrals, we obtain the following Adequate Boundary Condition (ABC):

$$p_t + c_0(1+M)p_z = -c_0 \sum_l \alpha \int_0^t Y_l(0,\tau) \frac{J_1[c_0(1-M)(t-\tau)\alpha]}{(t-\tau)} d\tau , \qquad (1)$$

where $\alpha^2 = \lambda^2(1+M)/(1-M)$, J is the Bessel function, $\lambda^2 = \lambda^2(l,m) = \beta^2 + m^2$, $\beta = \beta_l = (1/R)\xi_l$, with ξ_l being the l-th root of function $dJ_m(z)/dz$, m – the arbitrary positive integer, R – the outlet radius.

Similarly one can derive the ABC at inlet boundaries. Note, that inlet boundaries require specification of additional boundary conditions. One of them is represented by the standard equation of entropy conservation.

The ABC of Eq.(1) is essentially non-local both in space (summation over l) and in time (integration over τ). Numerical implementation of the operator in the RHS of Eq.(1) is a non-trivial task. Note, that at $R \to \infty$ the RHS of Eq.(1) vanishes, and we get a local boundary condition

$$p_t + c_0(1+M)p_z = 0\,, \quad (2)$$

which is often applied at finite R as an approximation of ABC [1].

RESULTS

Test calculations were made for simple 2D flow reactor and for combustors with conical and cylindrical flame holders. The flow in the test cases was modeled by (a) set of Euler equations, and (b) set of Navier-Stokes equations coupled with k-ε-model of turbulence and simple eddy break-up combustion model.

In the computational experiments we checked the transparency of inlet and outlet boundaries to pressure waves under various boundary conditions:

(I) Constant Pressure Boundary Conditions;
(II) von Neumann Boundary Conditions;
(III) One-dimensional Characteristic Based Boundary Condition of Eq.(2);
(IV) New Multi-Dimensional Boundary Condition of Eq.(1) with Convolution Integral.

It is shown that Boundary Conditions (I) and (II) show poor performance as to transparency to pressure waves. One-dimensional condition (2) is somewhat better, however some reflection is still evident. Application of the new multi-dimensional boundary condition (1) with the rough approximation of the convolution integral results in almost no pressure wave reflection.

1. *Grinstein F.J.* // Compt. Physics. 1994. 115. 1. P. 43.