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brightness of detonation in the substance. For a wave of insufficiently high strength, the luminosity due to detonation is absent. The luminosity recording unit is similar to that used in previous works for measuring shock front temperatures.

Apart from high resolution (with response times as short as a few nanoseconds) attained in recording the adiabatic explosion delay times by means of photomultipliers, other light detectors, and recording devices of modern design, the experimental scheme described above has the advantage that the initiating wave strength is monitored in each experimental run. High efficiency of the monitoring is ensured by the exponential scaling of shock front luminescence spectral brightness with temperature, whereas the temperature scales almost linearly with the pressure behind the shock front in the pressure range of interest for initiation of explosions of homogeneous (liquid or monocrystalline) samples.

An additional advantage of the scheme proposed here is the possibility of refinement of the shock adiabat for the explosive under study by measuring its luminescence brightness near the interface of its contact with the liquid, which would not require separate experiments. For example, using expressions for molecular crystals, with allowance being made for negligible value of change in both temperature, $\Delta T$, and specific volume across the interface, one has

$$\Delta T \propto -\frac{\xi_{12} \Gamma}{c_v} \Delta u_c,$$

where $c$ is the sound speed behind the wave front, and $\Delta u_p$ is the change in mass velocity across the interface. If the shock adiabat is written in the form $u_p = c_0 + bu_p - au_p^2$, then the value of $c$ is related to the mass velocity behind the front, $u$, by the expression

$$c^2 = \frac{(u_r-u_p)^2}{c_0 + au_p^2} \left[ c_0 + 2bu_p - 3au_p^2 - \xi_{12} \Gamma \frac{u_p^2}{u_{r}} \right].$$

The pressure change across the interface is

$$\Delta p = \rho_0 c \frac{\Delta u_p}{u_r - u_p},$$

where $\rho_0$ is the initial density of the liquid. In the experiments involving, instead of the substance studied, a window material with a known shock adiabat, the value of $\xi_{12} \Gamma$ is refined, to be used, subsequently, for determining the mass velocity at the contact interface by the change in luminescence brightness at the interface.

The technique has been validated in measurements of initiation limits for liquid nitroethers, azidoethanol, and a number of other compounds. It can be used in combination with other recording schemes, and modified with a view of obtaining additional data, such as detonation parameters of the substance under study.

MODELING OF CRITICAL CONDITIONS FOR DROPLET BREAKUP IN REACTIVE FLOWS

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INTRODUCTION. Deformation and breakup of liquid droplets are known to be the determining processes in a number of applied problems of multiphase flow, e.g. power engineering [1] and explosion safety [2]. Detailed reviews on the topic are published elsewhere [3,4].

In theoretical studies there exist two approaches. The first is based on the analysis of complex deformed state of a liquid droplet governed by multi-dimensional equations of motion. The
second is based on simplified models. The former approach succeeds mainly due to
development of computer technology. Application of simplified models of
droplet deformation and breakup
is justified when studying reactive multi-phase flows.

Proposed in [5] is a simplified model of droplet deformation based on the approximation
of 'Dominant Direction of Deformation' (DDD). The DDD approximation implies considering
a single-axis deformed state of a droplet of viscous incompressible liquid due to
dynamic pressure, surface tension and gravity forces. The model of [5] is based on the Stocks friction
law relating the normal stress to the rate of linear deformation of a particle. Quantitatively,
the model of [5] gives significantly underestimated values for the characteristic time of droplet
deformation and for the critical Weber number.

This paper deals with further improvement of the model of [5]. A main distinguishing
feature of our model is the attempt of taking into account internal viscous flows in a deformed
droplet. We use the model for predicting droplet behavior and the critical Weber number under
conditions of aerodynamic loading by a shock-induced gas flow.

FORMULATION. The initial droplet is assumed to be a sphere of radius $R$ while a
deformed droplet to have the shape of oblate ellipsoid of revolution elongated across the gas
flow. The ellipsoid is formed by revolution of ellipse with a short half-axis $a$ and long half-axis $b$
around the short axis. We assume also, that droplet deformation becomes irreversible when a
certain critical deformation with $a = a_\ast$, $b = b_\ast$ is attained. Let the direction of droplet motion
(X-axis) coincides with the direction of unconfined gas flow.

The origin of X-axis is placed in the center of symmetry of a droplet. Within the DDD
approximation the equation for linear droplet deformation along the X-axis can be written in
the following form:

$$2\mu_1 a^{-1} \frac{da}{dt} = -k \Delta p$$

where $t$ is time, $\mu_1$ is the dynamic viscosity of liquid, $\Delta p$ is the resultant deforming pressure,
$k < 1$ is the non-dimensional coefficient introduced here for taking into account the work done
by both aerodynamic and surface tension forces to generate internal motion of liquid inside a
droplet.

The resultant deforming pressure is defined as the difference between the resultant aerody-
namic pressure $p_d$ on the droplet surface and the reduced Laplace pressure $p_\sigma$, i.e. $\Delta p = p_d - p_\sigma$.
The latter is given by [5]:

$$p_\sigma = \frac{\sigma a^2 + b^2}{ba^2} - 2\sigma \frac{a}{b^2}.$$  \hspace{1cm} (2)

Determination of coefficient $k$ is based on considerations of the likelihood theory. Assuming
that droplet deformation is quasi-static one obtains: $k = f(u_1, \mu_1, R, \Delta p_\ast)$ where $u_1$ is the
characteristic velocity of liquid, $\Delta p_\ast$ is the characteristic value of the resultant deforming pressure.
Four governing parameters $u_1$, $\mu_1$, $R$, and $\Delta p_\ast$ allow one to construct a single independent
non-dimensional combination: $\frac{u_1 \mu_1}{R \Delta p_\ast}$. Therefore,

$$k = \alpha \frac{u_1 \mu_1}{R \Delta p_\ast}$$ \hspace{1cm} (3)

where $\alpha$ is the numerical coefficient. Clearly, the value of $u_1$ is of the same order as the rate
of droplet deformation $da/dt$. This allows estimating the order of the coefficient $\alpha$ by using
Eqs. (1) and (3): $\alpha \sim 1$.

For estimating $u_1$ one can use the continuity equation for a droplet of incompressible liquid.
Taking $u_1$ as a scaling value for a radial velocity component and $u_1 = u \left( \frac{\rho_0}{\mu_1 \rho_1} \right)^{1/3}$ [6] for angular
velocity components we come to a conclusion that \( u_1 \sim u_i \). Here \( u \) is the relative velocity of the center of symmetry of droplet in the gas flow. Replacing \( u_1 \) by \( u_i \), and assuming \( \Delta p_\ast = \frac{\rho u_i^2}{2} \), and \( \alpha \approx 1 \) one obtains from Eq. (3):

\[
k \approx \frac{2 \mu_1}{\rho u R} \left( \frac{\mu p}{\mu_1 p_1} \right)^{1/3}
\]  

(4)

The equation for \( u \) is a standard equation of motion:

\[
\frac{du}{dt} = -\frac{3}{4} \frac{C_x}{\alpha} \left( \frac{\rho u}{\rho_1} \right)^2
\]

(5)

where \( C_x \) is the aerodynamic drag coefficient. For blunt bodies like a deforming droplet, at high Reynolds numbers \( Re = \frac{\rho u R}{\mu} \) one can apply approximation

\[
C_x = \frac{p_d}{\rho u^2} = \frac{\bar{p}_d}{\bar{p}_d}
\]

(6)

where \( \bar{p}_d \) is the pressure coefficient.

Available experimental data on the drag coefficient of droplets undergoing deformation in compressible gas flow behind a shock wave indicate that \( C_x = 1.6–3.0 \) at \( Re \approx 500 \) and independent of \( Re \). In this paper we assume \( C_x = 2 \) = const, which is somewhat average of the experimental values.

Thus, we come to a conclusion that Eqs. (1) and (5) together with additional relationships (2), (4), (6) and with given values of \( \rho, \rho_1, \mu, \mu_1, R, \phi(t) \) define the problem of liquid droplet deformation within the DDD approximation.

For the sake of convenience we introduce dimensionless variables:

\[
t = \bar{t} \left( \frac{\mu_1 R}{\sigma} \right), \quad a = aR, \quad b = \bar{b}R, \quad u = \sqrt{\frac{\sigma}{\rho R}} W e^{1/2}, \quad We = \frac{\rho u^2 R}{\sigma}, \quad La = \frac{\rho \sigma R}{\mu_1^2},
\]

where \( We \) and \( La \) are respectively Weber and Laplace numbers. Then Eqs. (1) and (5) can be transformed as follows:

\[
\frac{d\bar{a}}{dt} = -\frac{1}{2} W e^{1/2} L a^{-1/2} \left( \frac{\mu}{\mu_1} \right)^{1/3} \left( \frac{\rho_1}{\rho} \right)^{1/6} \left[ C_x + \frac{2}{W e} (2 \bar{a}^3 - \bar{a}^{3/2} - \bar{a}^{-3/2}) \right]
\]

(7)

\[
\frac{dW e}{dt} = -\frac{3}{4} C_x \frac{W e^{3/2}}{\bar{a}} \left( \frac{\rho}{\rho_1} \right)^{1/2} L a^{-1/2} + 2 \phi' W e^{1/2}
\]

Initial conditions are:

\[
\bar{t} = 0, \quad \bar{a} = 1, \quad W e = W e_0 = \frac{\rho u^2(0) R}{\sigma}.
\]

A minimum Weber number \( W e_\ast \) required for attaining a critical deformation stage \((\bar{a}_\ast, \bar{b}_\ast)\) we call the critical Weber number. For determining \( \bar{a}_\ast \) we use the following consideration. Numerous experimental observations of droplet breakup behind shock waves indicate that for droplets of low-viscosity liquids (water, kerosene, alcohol) of relatively large size (1 to 4 mm)
We \approx 5. At such conditions \( \Lambda = 10^3 - 10^5 \). Since \( \frac{\rho}{\mu} = 10^2 - 10^3 \), then during droplet deformation Eqs.(7) give \( \frac{dW_e}{dt} \ll 1 \), i.e. \( W_e \approx W_e_0 \approx \text{const.} \). Reversible droplet deformation implies the existence of the stage with \( \frac{d\bar{a}}{dt} = 0 \), or by using Eqs.(7)

\[
\frac{C_2 W_e_0}{2} = \frac{\bar{a}^{-3/2} + \bar{a}^{3/2}}{2} - 2\bar{a}^3
\]

A critical stage for droplets of low-viscosity liquids is attained at \( W_e_0 = W_e_0* = W_e* \approx 5 \), and hence at \( \bar{a}_* \approx 0.35, \bar{b}_* \approx 1.7 \).

Figure 1. A comparison of predicted dependency \( W_e*(\Pi) \) (solid curve) with available experimental data (points). The range \( 0 < \Pi < 0.5 \) corresponds to realistic liquids (mercury, water, alcohols, kerosene, oils, glycerine, etc.).

RESULTS. The set of ordinary differential equations (7) has been solved numerically by Runge-Kutta method of the 4th order with variable time step. The model has been validated by comprehensive comparisons between predicted results and available experimental data on droplet deformation and breakup behind shock waves. Comparisons were made for ‘subcritical’, ‘critical’ and ‘supercritical’ modes of droplet loading. It is shown, that our model gives rather satisfactory results for droplet deformation dynamics and for the effect of various governing parameters on it.

Of most interest are the data on the ‘critical’ mode of droplet loading. The integral curve in the \( (W_e, \bar{a}) \)-plane is given by the differential equation

\[
\frac{dW_e}{d\bar{a}} = \frac{3}{2} C_2 \left( \frac{W_e}{\bar{a}} \right)^{2/3} \left( \frac{\rho}{\mu} \right)^{2/3} \left( \frac{\mu}{\mu} \right)^{1/3} \left[ C_2 + \frac{2}{W_e} (2\bar{a}^3 - \bar{a}^{3/2} - \bar{a}^{3/2}) \right]^{-1}
\]
with the initial condition $\bar{a} = 1$, $We = We_0$. Therefore,

$$We_{0*} = f \left( \bar{a}_*, \left[ \left( \frac{\rho}{\rho_l} \right)^{2/3} \left( \frac{\mu_0}{\mu} \right)^{1/3} \right] \right)$$

i.e. at a given $\bar{a}_*$ the critical Weber number $We_{0*}$ is dependent on a single dimensionless parameter $\Pi = \left( \frac{\rho}{\rho_l} \right)^{2/3} \left( \frac{\mu_0}{\mu} \right)^{1/3}$. It is important that the Laplace number does not belong to the list of governing parameters for $We_{0*}$.

Figure 1 shows the predicted dependence $We_{0*}(\Pi)$. Calculations were made for both pure liquids (mercury, water, alcohols, kerosene, oils, glycerine, etc.) and mixtures (water + glycerine) at incident shock Mach numbers $1 < M < 1.2$, For comparison, experimental points of [7–11,4] are plotted in Fig.1. The reason for the discrepancy in data of [10] in Fig.1 is that the authors of [10] give only the ranges of $R$ and $M$ used in their experimental study, as well as calculated Laplace numbers, instead of presenting the data for any particular observation. Discrepancy in data of [7,4] is due to distinguishing the lower and the upper thresholds of droplet stability in [7,4]. Worth noting are the data of [9] in Fig.1, which indicate a significant dependence of $We_{0*}$ on droplet size. Our attempts to find experimental data within the ranges $0.2 < \Pi < 0.4$ and $\Pi > 0.45$ have failed. In general, there is a need in further experimental studies for making conclusions on the validity of the model concerning conditions for critical droplet loading.

References


