Calculation of the Velocity of Gaseous Detonation in a Rough Tube Based on Measurements of Shock Wave Attenuation

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Abstract

When considering the mechanism of gaseous quasidetonation in rough tubes, one introduces drag and heat transfer coefficients into governing equations describing unsteady combustible gas flow with shock waves. These quantities are usually taken from steady unreacted flow measurements in the form of empirical correlations. Since quasidetonation is a topic of considerable practical importance, one needs a detailed, quantitative theoretical and experimental substantiation of the approach. For this purpose we have developed a simple approximate analytical theory to predict the rate of attenuation of a planar shock wave propagating through a gas in a tube with rough walls. It involves the following basic assumptions: 1) the flow is one-dimensional; 2) heat transfer to the walls is neglected; and 3) shock wave velocity is independent of conditions behind it (this is known as Whitham’s approach) and depends only on tube geometry, distance traveled by the shock wave in a section with obstacles, and its initial Mach number.

Comparison of the solution with detailed numerical studies has been made, and good agreement between predicted results is shown to exist. Systematic measurements of shock wave...
attenuation in tubes with obstacles made of aluminum rings have been carried out to verify the theory. The data were used to determine momentum losses in shock waves by means of fitting the measured and predicted results. The drag coefficients thus obtained are shown to be larger by a factor of about four than the corresponding values measured in steady flow conditions within the whole experimental range of roughness configurations. Finally, we apply the results for calculating quasidetonation velocity as a function of ring height and spacing.

Introduction

In principle, one should consider any real detonation wave as nonideal since it is well known that the Zel'dovich-Neumann-Doring model of detonation in its one-dimensional form is not realized in practice. However, in this study the term "nonideal detonation" implies a detonation wave characterized by momentum and energy losses due to interaction of the gas with wall roughness. Taking into account this interaction is of great practical importance because it allows for prediction of the behavior of detonating systems under various conditions and explains why their characteristics are different in laboratory-scale and real, large-scale conditions. Extensive experience in studying gaseous mixtures indicates that momentum and energy losses in detonation waves should be taken into account along with the nonsteady nature of the shock wave-reaction zone complex and the cellular structure of detonation. Clearly, even such highly detonable mixtures as $\text{C}_2\text{H}_2 + \text{O}_2$ and $2\text{H}_2 + \text{O}_2$ have, in tubes of sufficiently small diameter, detonation velocities somewhat lower than those calculated without allowance for losses.

The losses markedly modify the interrelation of such factors as multidimensionality, the nonsteady nature of the flowfield behind the detonation wave front, and the total losses responsible for detonation limits; they also decrease the detonation velocity. This modification may turn out to be so dramatic that new regimes of detonation propagation come into effect and considerable difference may be observed
in the wave structure, compared to the standard CJ case. Roughness elements on the channel wall disturb the flowfield, distort the regular three-dimensional, nonsteady pattern of the detonation cells, and generate additional hot spots that make the reaction proceed in a more stable fashion. This indicates that one-sided approaches, using either stability criteria of the front and the reaction zone without regard for losses or one-dimensional models accounting for the effect of losses, are inadequate for interpreting the detonation process. Unfortunately, a comprehensive analysis of the multidimensional flow in detonation waves is not possible at present. A rather detailed qualitative generalization of quasidetonations in tubes with obstacles has been recently suggested.

Since a quantitative analysis of the multidimensional flow behind a detonation wave is extremely complex, even for modern computers, it is worth considering possible variations of a one-dimensional theory, compared to the standard CJ theory, when taking momentum losses into account. The one-dimensional approach has already demonstrated its benefit for treating not only stationary waves but also such an essentially multidimensional and nonsteady phenomenon as detonation initiation. Solution of the governing equations for the average quantities behind the detonation front with properly chosen drag and heat coefficients may be expected to give not only qualitatively but also quantitatively correct information on the average nonideal detonation velocity under real conditions. That is, it can serve as a basis for engineering estimates and future multidimensional theories.

When considering gaseous detonation in rough tubes, one introduces into the governing equations the drag and heat transfer coefficients that are usually taken from steady unreacted flow measurements in the form of empirical correlations. It is pointed out that this approach underestimates considerably the momentum losses in the reaction zone of a nonideal detonation wave. Some modifications of the empirical correlations are proposed for the drag coefficient. It appears that in the one-dimensional theory the local drag coefficient is
infinite just behind the detonation front and drops downstream of the reaction zone to a certain, almost constant value. Depending on the length of the reaction zone, the average drag coefficient in it may become considerably larger than that determined under the steady flow conditions.

The present study deals with further development of the question. We propose a simple approximate theory of shock wave attenuation in a rough tube, including an a priori unknown drag coefficient. In order to determine the latter, we have carried out experiments in a shock tube equipped with properly designed roughness and fitted the experimental and theoretical curves of shock wave attenuation along the tube. The drag coefficient thus obtained is used in the sequel for calculating nonideal detonation velocity.

Mathematical Modeling of Shock Wave Attenuation in a Rough Tube

Consider propagation of a shock wave through a tube with roughness elements distributed uniformly on the internal tube surface. The obstacles interact with the flow behind the shock front, exerting dynamic influence on it and distorting its spatial structure. Laser schlieren photographs and density distribution measurements indicate that behind the shock wave front a regular wave structure appears, the shock front becomes curved to a certain degree, and the density is distributed almost uniformly across the tube except for a narrow layer attached to the wall. One may believe that these findings justify the one-dimensional approach to the problem of shock wave attenuation.

Interaction of the flow with roughness elements results both in momentum losses and kinetic energy transformation owing to bow shock formation and turbulence generation and dissipation. Heat transfer to tube walls and roughness elements is relatively insignificant in the total energy balance. Experimental studies of steady stabilized flows in tubes with rough walls reveal the existence of a quadratic drag law for large Reynolds numbers $Re$. By analogy we assume
from now on that the drag coefficient in the nonsteady flow behind a shock wave depends only on obstacle geometry and is independent of the Reynolds and Mach numbers of the flow.

Formulation

Let us consider evolution of a stepwise shock after its entry into a rough section \( 0 < x < \infty \) of a tube of radius \( r \). The shock wave is assumed to travel initially along the smooth section \( -\infty < x < 0 \) of the tube at a constant velocity. On the assumptions formulated previously, the governing equations for perfect gas flow are as follows:

\[
\begin{align*}
\rho_t + u\rho_x + \rho u_x &= 0 \quad (1a) \\
\rho u_t + \rho u u_x + p_x &= -F \quad (1b) \\
p_t + up_x - a^2 (\rho_t + u \rho_x) &= (\gamma - 1) Fu \quad (1c) \\
p &= \rho RT \quad (1d)
\end{align*}
\]

where \( \rho \) is the density, \( u \) is the velocity, \( p \) is the pressure, \( a \) is the sound velocity, \( T \) is the temperature, \( R \) is the gas constant, \( \gamma \) is the ratio of specific heats, and indices \( t \) and \( x \) denote derivatives in time and coordinates. In the smooth section \( F=0 \), whereas in the rough section \( F = \frac{(C_f/r)}{\rho u|u|} \), where \( C_f \) is the local drag coefficient referred to henceforth to as the coefficient of momentum losses (in order to distinguish it from the drag coefficient \( C_f' \)) in a steady stabilized flow.

The initial conditions at the rough section inlet are

\[
\begin{align*}
t &= 0, -\infty < x < 0, \quad (2a) \\
M &= M_0 \quad (2b) \\
\gamma + 1 \quad (2c) \\
\rho &= \rho_0 \frac{(\gamma + 1)}{(\gamma - 1) M^2 + 2}, \quad (2d) \\
u &= \frac{2}{\gamma + 1} \frac{a_o}{M} \quad (2d)
\end{align*}
\]
\[ p = p_0 \left[ \frac{2\gamma M^2}{\gamma+1} \right] \]  \hspace{1cm} (2e)

and

\[ t = 0, \; 0 < x < \infty \]  \hspace{1cm} (3a)

\[ \rho = \rho_0, \]  \hspace{1cm} (3b)

\[ u = 0, \]  \hspace{1cm} (3c)

\[ p = p_0 \]  \hspace{1cm} (3d)

where \( M_0 \) is the incident shock wave Mach number and index 0 labels the quantities in the unperturbed flowfield. At any instant the conditions given by Eqs. (2a)-(2e) hold for \( x \to -\infty \) and those given by Eqs. (3) hold for \( x = \infty \).

**Results of the Analysis**

In spite of a number of simplifications introduced in Eqs. (1a)-(1d), their exact solution as far as we know is not available at present. In most of the cases considered, the problems involving nonlinear interaction of a shock wave with confinement walls are treated only approximately. Quite successful in this sense is the approximate approach suggested by Whitham. This phenomenological approach is based on a simple assumption that has no fundamental substantiation to date. Following Ref. 9 we assume that the perturbed flow behind the shock wave exerts no influence on shock wave propagation; that is, variations in the shock wave velocity are solely due to interaction of the front itself with the obstacles. The method discussed consists in substitution of Eqs. (2a)-(2c) into the equation for \( \chi_+ \)-- characteristics derived from Eqs. (1a)-(1d). The solution of the ordinary differential equation thus obtained assumes the form

\[ G(m_0) - G(m) = C f x/r \]  \hspace{1cm} (4)
In the linear approximation \( m = M - 1 \), \( m_0 = M_0 - 1 \), and \( G(M) = -(\gamma + 1) \gamma 2m \). For the case of arbitrary initial shock wave Mach number \( m = M \), \( m_0 = M_0 \) and

\[
G(m) = \frac{0.4m - 1}{m^2 - 1} + 4 \ln(m^2 - 1) + 0.8 \ln \frac{m + 1}{m - 1} \tag{5}
\]

Equation (5) is an approximation of the exact function \( G(M) \). Deviation of the results given by Eq. (5) from the exact solution of Eq. (4) is less than 5% in the range \( 1.01 < m < 4 \).

In order to determine the limits of the validity of Eq. (4), numerical integration of the set of Eqs. (1a)-(1d) was carried out by the method outlined in Ref. 10. Figure 1 shows the results of numerical calculations in the form of dependence of the shock overpressure \( \Delta p_f \) on the dimensionless complex \( C_f x/r \) for three different initial shock Mach numbers. Solid lines represent the numerical solution; the dotted and dashed lines represent respectively the linear approximation of Eq. (4) and the approximate solution given by Eqs. (4), and respectively, (5). A comparison of the results indicates that for \( M_0 < \)

![Fig. 1 Predicted shock wave attenuation in a rough tube. 1-\( M_0 = 1.877 \), 2-1.497, 3-1.272. Solid line is approximate solution given by Eq. (4), dashed line is linear approximation, and dotted line is numerical simulation.](image-url)
Fig. 2 Experimental setup.

Fig. 3 Obstacle configuration.

Fig. 4 Typical pressure-time histories in a shock wave propagating through a rough tube.
1.5 there exists a good agreement (less than 10% deviation) of linear approximation and the solution based on Eqs. (4), and (5) with the results of numerical calculations.

An increase in $M_0$ deteriorates the consistency of the predictions based on linear approximation; however, the accuracy of approximation given by Eqs. (4), and (5) remains quite satisfactory in a wide range of $C_f x/r$ values. This allows us to put forward a concept that Eqs. (4), and (5) are applicable to approximate calculations of the overpressure in a shock wave propagating through a rough tube. Equation (4) contains a priori the unknown coefficient $C_f$. In order to determine it, an experimental study of shock wave attenuation in a tube with rough walls was undertaken.

Experimental

A sketch of the experimental setup is shown in Fig. 2. We used a 50-mm-diameter steel vertical shock tube, consisting of a high-pressure chamber (I) 0.5-m long and a low-pressure chamber (II) 1.5-m long. An aluminum ring assembly (see Fig. 3) properly inserted in a low-pressure chamber created obstacles in the rough section III. Six flush-mounted piezoelectric pressure transducers were used to register shock front overpressure and pressure histories (see Fig. 4),
as well as to measure the shock wave velocity. In the experiments the quantities $M_0$, ring spacing $s$, and ring height $k$ have been varied. Rings with $k = 5, 10 \text{ mm}$ and $s = 8, 13, 28, 53, 78, 103, \text{ and } 128 \text{ mm}$ were used in the experiments. The unknown coefficient of momentum losses $C_f$, pertaining to a given obstacle configuration (specified by values of $k$ and $s$), was determined on the basis of a comparison of measured attenuation curves (see Fig. 5) and the approximate law given by Eqs. (4), and (5). For this purpose the optimal value of $C_f$ for each run was first determined by the least-squares method. Then for a given obstacle configuration the average value $C_f (k, s)$ in the range $1.1 \leq M_0 \leq 2$ was derived.

The experimental points thus obtained are shown in Fig. 6. The ordinates of the points correspond to measured values of $\Delta p_f$. The abscissa of each point is calculated by the use of the averaged value of $C_f (k, s)$ and the distance $x$ between the inlet of the rough section and the proper pressure transducer. As follows from Fig. 6, the assumption that $C_f$ is independent of $M_0$ and $Re$ numbers agrees satisfactorily with the experimental results since the discrepancy between the points varies insignificantly with variations.

![Fig. 6 A comparison of the predicted (solid lines) and measured (points) results on shock wave attenuation in a rough tube. (1) $M_0 = 1.877$; (2) 1.497; (3) 1.272.](image-url)
of $\Delta p_f$ and obstacle configuration. The experimental points corresponding to essentially different levels of losses are grouped close to the theoretical curves. This is evidence in favor of the fact that the parameters $M_0$ and $C_f x/r$ are indeed the quantities governing the process of shock wave attenuation.

Figure 7 shows the dependence of $C_f$ on $k$ (1–k=5 mm, 2–10 mm) and $s$ ($s = s^{-1}$) (solid lines). Clearly, the dependence is nonmonotonic. If $s/k = 5–10$, the coefficient $C_f$ attains its maximum value; that is, shock wave attenuation is the most efficient in this case. For a quantitative comparison of the coefficient of momentum losses $C_f$, with the hydraulic drag coefficient $\lambda$ measured in a steady stabilized flow, the data by Koch were used (dashed lines). In the quoted study the tube diameter (50 mm) and obstacle configurations (assemblies of thin rings) were the same as those employed in our study. Therefore, a direct comparison of results is justified. It follows from the comparison of solid and dashed lines in Fig. 6 that the coefficient of momentum losses in the nonsteady flow behind a shock wave correlates unexpectedly well with the hydraulic drag coefficient $\lambda$ measured in conditions of a steady stabilized flow. Note that the one-dimensional

![Figure 7](image-url)
theory of a steady flow operates with the quantity $C_f' = \lambda / 4$. Hence the coefficient of momentum losses $C_f$ in the flow behind a shock wave is approximately four times larger than $C_f'$. This conclusion confirms the theoretical prediction dealing with the drag coefficient behind a nonideal detonation wave.

Nonideal Detonation

The dependence $C_f(k, s)$ (see Fig. 7) permitted us to suggest a complete one-dimensional theory of gaseous detonation in a rough tube, and it showed a way purposefully to vary the detonation velocity and suppress detonation. Application of the available models seems to produce a reasonable explanation of some experimental observations.

Figure 8 shows the calculated dependence of the detonation velocity on ring spacing ($n_B = s^{-1}$), drawn with due regard for the results and the data of Fig. 7 ($D_0$ is the CJ detonation velocity). The curves represent only preliminary results and are based on model chemical kinetics having no relation to any particular detonable mixture.

Curve 1 corresponds to the detonation regime with combustion in a boundary layer in a 50-mm-diameter tube with a ring height $k$ of 5 mm. Curve 2 corresponds to the detonation regime with the bulk ignition behind the front in a tube of the same diameter but $k = 10$ mm. In the latter case the kinetic parameters of the model chemical reaction

![Fig. 8 Predicted dependence of the detonation velocity of a model detonable mixture on ring spacing. Curve 1 corresponds to the detonation regime with boundary layer combustion; curve 2 corresponds to the detonation regime with bulk ignition.](image-url)
Fig. 9 Measured dependences of the detonation velocity of C_2H_2 + O_2 mixture in a rough tube: (1) d=9 mm, k=1 mm; (2) d=7 mm, k=1 mm; (3) d=7 mm, k=1.6 mm.

with a rate constant of the Arrhenius type are as follows: activation energy 2 MJ/kg, heat release 2 MJ/kg, preexponential factor 10^{10} s^{-1}, and reaction order 1. The D/D_0 (n) curves have a strongly pronounced minimum. Qualitatively, the similar dependences of D on n_0 are obtained in Ref. 11 (see Fig. 9). Thus, variation of obstacle geometry allows one to control the detonation velocity. The minimum dependence of detonation velocity on obstacle spacing observed experimentally seems to be due to the fact that the hydraulic drag coefficient λ attains its maximum value. It is of importance that under the purposeful variation of the detonation velocity by means of a proper choice of the roughness configuration, one may use available data on hydraulic characteristics of channels with different roughness. This considerably widens the possible engineering solutions for safety measures aimed at mitigating detonations.

Conclusion

Our study of shock wave attenuation in rough tubes reveals the following features. The drag coefficient in nonsteady flow conditions behind a shock wave correlates satisfactorily with C, the hydraulic drag coefficient, measured in steady, stabilized flow conditions. The principal parameters governing shock wave attenuation are
the initial Mach number and the dimensionless complex $C_r x/r$. The approximate law of shock wave decay given by Eq. (4) agrees satisfactorily with the numerical calculations. The configuration of wall roughness with $s/k = 5-10$ produces a maximum drag and consequently provides the most efficient shock wave attenuation. The results obtained allow calculating the velocity of nonideal detonation in a rough tube within the framework of one-dimensional theories.

References


8Koch, R., Loss of Pressure and Dissipation of Heat by (verwitzbelter) Electricity, VDI-Forsch.-Heft, 1958, p. 469.
